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UDC 514.764.2

Conformally flat splines

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The interior of the unit ball corresponds to the Lobachevsky space in the Klein's model. Convex subsets in Lobachevsky space coincide with the usual convex subsets of the unit ball. However, the convex geometry of Lobachevskii space is more substantial in analytical plan.

In particular, one can naturally associate an arbitrary compact convex subset Q with conformally flat metric $ds^2 = \frac{dx^2}{h_Q^2(x)}$, $x \in R^{n-1}$, of bounded one-dimensional sectional curvature $K(x,\xi)$, which is defined on $\overline{R^{n-1}}$ [1]:

$$-\frac{\kappa}{2} \le K(x,\xi) = h_Q \frac{d^2 h_Q}{d\xi^2} - \frac{1}{2} |\nabla h_Q|^2 \le \frac{\kappa}{2}, \qquad (1)$$

where $h_Q(x)$ is a positive function of the class $C^{1,1}$ defining conformally flat metric, ∇h_Q is the gradient of $h_Q(x)$ satisfying to the Lipschitz's condition, $\frac{d^2 h_Q}{d\xi^2}$ is the second derivative of function h_Q, in sense F. Clark [2], along arbitrary unit vector $\xi \in R^{n-1}$, $\kappa > 0$ is a positive constant such that $(-\kappa)$ is the curvature of Lobachevsky space.

In this paper we call such functions as support functions for a convex set Q (see also [3-4]). In the case of a finite convex polyhedron of Lobachevsky space the following formula is true

$$h_Q(x) = \min_i \left\{ h_{\Delta_i}(x) \right\}, \qquad (2)$$

where $h_{\Delta_i}(x)$ are support functions of $(n-1)$ dimensional sides of the border of Q.

Calculation of functions $h_{\Delta_i}(x)$ occurs recurrently and reduced to the case when Δ_i are k-dimensional simpleksa ($k < n$). Such functions will be called elementary conformal splines [3].

In contrast to the conventional presentation of spline functions, the representation (2) of the function $h_Q(x)$ via conformally flat spline functions has another nature, since not required to specify the domain of definition $h_{\Delta_i}(x)$. Function $h_Q(x)$ has smoothness $C^{1,1}$, and any function $f \in C^1$ can be arbitrarily closely approximated via function $h_Q(x)$ of the form (2) in the norm of C^1 – space on a compact subset (for sufficiently large κ). The obvious formula (2) for the function $h_Q(x)$ allows us to simplify calculation and to make its more effective: it isn't need to break the domain of definition of $h_Q(x)$, and it is possible to use parallel algorithms for calculation of elementary splines. In work [4] the algorithm of calculation is realized in MatLab and Mathematica packages.

In a one-dimensional case the graph of function $h_Q(x) = \min_{i=1,..,4} \left\{ h_{\Delta_i}(x) \right\}$ composed of four one-dimensional splines is represented in figure 1. Each spline $h_{\Delta_i}(x)$ corresponds to the segment Δ_i in Lobachevsky's plane (see figure (2)).

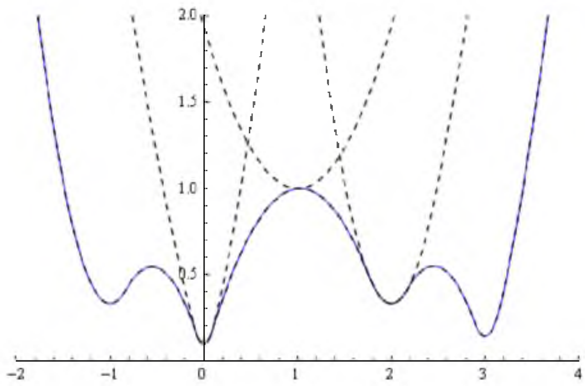

Fig. 1


Fig. 2

These investigations are supported by the grants: the Government of the Russian Federation (#14.B25.31.0029, Nsh of RF (#2263.2014.1)), the RFBR (15-41-00092r -the Urals, 15-41-00063 r-the Urals, 15-01-06582-a, 16-01-00336-a)), the Russian Federation Ministry of Education (Project Code: 1148).

In concluding we mention some works about other problems in the theory of Riemannian spaces [6−10].

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